# Risk Management in a Bayesian Financial Model

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## ABSTRACT

In this paper, we provide an outline of risk management techniques used in conjunction with the Bayesian methodology that is applied to forecast financial time series data. The paper presents three main sections. Section 1 provides an introduction. Section 2 gives an overview of the classical Mean Variance Optimization (MVO) methods that form a starting point for risk assessment and risk management. Section 2 also shows how the Bayesian methods are used to generate inputs to the MVO algorithm. Section 3 describes modifications and enhancements to the classical MVO methods that have been developed for use with our Bayesian time series forecasting model.

## KEYWORDS

Risk, Risk Management, GARCH, Principal Component Analysis, mean variance optimization, covariance matrix

## 1. INTRODUCTION

Management of risk is a fundamental part of an investment strategy. Earning returns and controlling risk are integral parts of the same process. Risk management requires understanding the sources of risk and taking the steps necessary to mitigate these sources of risk, while balancing the risk against the opportunities to profit.

There are many factors that affect market risk in the foreign exchange forecasting. There are the sensitivities to market and macroeconomic factors, such as country-specific interest rates and economic activity levels; there is the volatile level of risk perception among investors, and there are unannounced government interventions and unscheduled economic announcements or decisions.

Diversification of the investment portfolio over a set of disparate currencies helps to mitigate risk. Distributing the investment strategy over different driving factors also helps to mitigate risk. To balance the risk against opportunities, a variation of the classical mean variance optimization (MVO) procedure developed by Markowitz[1] is used to optimally distribute the assets in the portfolio. The goal is to optimally distribute the portfolio in such a way that the different currencies hedge risks from each other while maximizing the opportunities to profit from the investments.

## 2. CLASSICAL MEAN VARIANCE OPTIMIZATION

Classical MVO methods seek to optimize the return for a given risk primarily through diversification. The goal of this diversification is to distribute the assets optimally to avoid excessive exposure to a single asset or group of assets that are correlated and move together. Markowitz's MVO procedure gives a framework and methodology to assemble an optimal distribution of the assets in the portfolio. MVO methods calculate this optimal distribution using an estimate of the variance of each asset together with an estimate of the covariance of each asset with respect to each other asset. These estimates are assembled in the form of a covariance matrix. The information in this covariance matrix is equivalent to, and can be alternately expressed as, the set of volatilities of each asset together with the set of correlations between each pair of assets. This information is used to generate a portfolio in which the currencies are diversified and hedge risks from each other while maximizing the opportunities to profit from the investments.

The classical MVO provides a framework and starting point for risk management, which we enhance for use with our Bayesian model for currency exchange as detailed below. This balancing of risk vs. return requires a method to forecast estimates of returns and risk. These are discussed in the next two sections.

### 2.1 BAYESIAN GENERATION OF INPUTS

The inputs to the optimization procedure are based on the investor's beliefs about risk and return in the upcoming period. The optimization is integrated into the Bayesian strategy by using the results generated in the learning process of the Bayesian Dynamic Linear Model[2,3]. The expected returns are directly generated by the Bayesian Dynamic Linear Model. The expected variance and covariance of the investments in the portfolio are generated using the returns predicted by the Dynamic Linear Model predictions in conjunction with the actual returns, consistent with the Bayesian learning process. The historical errors collected in each time period provide a basis for calculation of the expected variance and covariance of the investments using GARCH methods and Principal Components Analysis described below.

## 2.2 GARCH METHODS AND PRINCIPAL COMPONENTS ANALYSIS

Generalized autoregressive conditional heteroskedasticity (GARCH) methods are well known to be good predictors for estimating the volatility of assets. GARCH methods correctly reproduce the empirical phenomenon observed in financial assets in which the volatility of a given asset is autocorrelated in time. In other words, the level of volatility of a given asset in the recent past can be a good indicator of volatility of that same asset in the near future.

Whereas standard, univariate GARCH methods are commonly used to predict the volatility of a single asset, the method below describes the optimal use of a multivariate GARCH method to predict the variance and covariance of a group of assets. This approach exploits some of the same principles of information pooling used in predicting returns[4].

A principal components analysis (PCA) is applied to approximately 20 years of historical error signals supplied by the Bayesian Dynamic Linear Model. This error signal represents variance of the market data with respect to the modeled returns, which is more pertinent to the analysis than the variance of the raw market data. The PCA generates the principal components of the error signal, which correspond to the orthogonal eigenvectors of the error signal. Each principal component contains a set of currency weights that defines the composition of a basket of currencies that accounts for a given portion of the total variance.

Because the principal components are orthogonal, each component is independent, and a univariate GARCH analysis can be independently applied to each component to predict the future variance of that component. When the variance of each of the components has been predicted, these predictions of the variance of each principal components is mapped back to the weighted individual currencies that comprise the principal components, resulting in the full variance-covariance matrix that defines the expected volatility of each asset along with its correlation with each other asset.

Thus, historical volatility information is pooled, and this pooled information generates an adaptive, time-varying variance-covariance matrix that provides a signal used by the optimization procedure to mitigate risk. This system identifies and reduces risk through diversification against excessive exposures to individual currencies as well as reducing systemic risks from exposures to baskets of currencies that move together.

## 2.3 IMPLEMENTATION

The above methods define a covariance matrix $\Sigma$. The Bayesian time series analysis defines an expected return vector $\alpha$. The portfolio manager specifies a target portfolio volatility $\sigma_0$.

Applying Lagrange multipliers to maximize the total portfolio return $w^T\alpha$ under the constraint that the target portfolio volatility $w^T\Sigma w$ is limited to $\sigma_0$ results in the set of investment weights w equal to:

$$w := \frac{\sigma_0}{\sqrt{\alpha^T \cdot \Sigma^{-1} \alpha}} \cdot \Sigma^{-1} \cdot \alpha$$

where:

- w is the weights vector for currency exposures;
- $\sigma_0$ is the target portfolio volatility;
- $\Sigma$ is the covariance matrix;
- $\Sigma^{-1}$ is the inverted covariance matrix;
- $\alpha$ is the expected return vector.

# 3. REFINEMENTS TO THE OPTIMIZATION PROCEDURE

## 3.1 TRANSACTION COSTS

A customized mean variance optimization procedure was implemented at P/E Investments to include the transaction costs in the optimization procedure. This has the effect of limiting spurious trades whose benefits are exceeded by the cost of executing the trades.

## 3.2 LIMITING THE SHARPE ANGLE

A technique developed by Golts and Jones[5] was implemented to overcome a limitation of the classic MVO procedure. The evolving covariance matrix is calculated from estimates of correlations that are based on historical data. During some periods, this procedure yields a set of correlation coefficients that result in over-reliance on the accuracy of these correlation coefficients and investments that are not aligned with the expected returns.

The expected portfolio return R is the product of the weight (exposure) of each asset in the portfolio times the respective expected return of that asset. This can be expressed as a dot product of the two vectors: $R = w \cdot \alpha$. The dot product also has a geometric interpretation which can be useful in interpreting the portfolio exposures in an intuitive way. From a geometrical perspective, the dot product is the product of magnitude of the two vectors times the cosine of the angle between the two vectors. This angle is called the Sharpe angle. When the Sharpe angle is high (near 90 degrees), the investment is not well aligned with the direction of return and the returns are highly sensitive to small errors in correlation coefficients.

This can be illustrated in a simple example. In Figure 1 below, the results from a naïve MVO are plotted for a two asset portfolio composed of the Euro (EUR) and the Swiss Franc (CHF). This example is assumed to be after a period in which these two currencies have been very highly correlated, with the EUR having a slightly higher expected return than the CHF. Accordingly, given the very high correlation, the MVO constructs a portfolio to exploit this nearly perfect arbitrage opportunity by investing in a long position in the EUR, nearly perfectly hedged by an equal short position in the CHF.

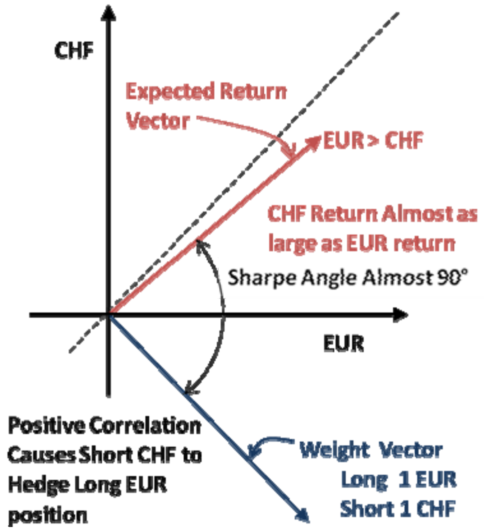


Figure 1 2D Plot for 2 Assets: EUR and CHF

What's wrong with this picture? Plotting the weights vector and the expected return vector on the same plot in Figure 1 shows that the Sharpe angle (S) is near 90º. In other words, the investment weights vector is orientated in a completely different direction from the expected returns vector. With S near 90º, cos(S) is near zero, so to compensate to maintain a high return R in the equation $R = |w|*|\alpha|*\cos(S)$, the weights w need to be leveraged up, greatly increasing exposures and risk. Furthermore, even slight changes in the correlation coefficient between the EUR and CHF can have drastic changes in the returns. For example, if the appreciation of the CHF were slightly higher, making it more than that of the Euro, the resulting Sharpe angle would be greater than 90º, making the cos(S) less than zero, yielding negative returns.

In order to account for uncertainties in the correlation coefficient, and to avoid what has been called "leveraging noise", limitations are imposed on the reliance on the values of these correlation coefficients. Thus, the Sharpe angle can function as a "canary in a coal mine" and provides a signal used to limit the allowable values of correlation coefficients and to limit exposures and tail risk.

### 3.3 EFFECTS OF LIMITING THE SHARPE ANGLE

Table 1 illustrates the effects of applying the Sharpe angle tool to the full FX model. The model includes the US Dollar and 9 other developed market currencies and is driven by four factors. The results in Table 1 show that limiting the maximum Sharpe Angle to 45 degrees reduces the average exposures in highly correlated assets such as the EUR and CHF approximately 45%-50% and reduces the average exposure of all of the assets in the portfolio approximately 24%. The reduction in exposures also reduces the maximum drawdown by 1.7% and has only a minimal impact (< 1%) on the overall information ratio.

| Sharpe | Average Exposure | | | Lifetime |
|---|---|---|---|---|
| Angle | EUR | CHF | Portfolio | IR |
| Baseline | 0.59 | 0.63 | 0.494 | 1.20 |
| 45° | 0.31 | 0.35 | 0.375 | 1.21 |

Table 1

This example shows how the Sharpe angle method maintains the portfolio's performance while reducing the exposures to unforeseen events and their associated tail risk.

## 4. CONCLUSION

This paper outlines classical optimization and risk mitigation techniques and shows how modified versions of these techniques are used to mitigate risk in the Bayesian FX modeling framework. Section 1 gives an introduction. Section 2 describes the classical MVO method and how the Bayesian methods are used to generate inputs. Section 3 describes refinements to the MVO method that improve its performance and robustness and limit exposure to tail risk.

## 5. REFERENCES


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